Rithmomachia: The Lost Mathematical Treasure of the Dark Ages

An Honors Thesis (HONRS 499)

by

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Abstract

Rithmomachia is an ancient game composed mainly of mathematical principles and positional strategies similar to that of Chess. Since there are a number of variants of the game, this discussion focuses on the most common rules and victories which have emerged throughout history. The latter half of the discussion pertains to the analysis of twelve different games of Rithmomachia where players ranged from eleven to fifty years old. This direct observation is a further attempt to replicate the intimate relationship between the fascination of numbers and the study of mathematics.

Rithmomachia is a tournament worthy of intellectual foes. Early combatants were in fact trained in Boethius's number theory, Latinized Nicomachus, and the Pythagorean philosophy of numbers. It was a game for the elite and not the general public. This very noble and ancient Pythagorean game amused scholars as they sought to understand the true significance of numbers. During its six hundred years of life, many marveled at the power of numbers to fascinate the human mind. The game's popularity, however, began to waver at the close of the seventeenth century and then simply disappeared for over seven hundred years. Many historians have tried to uncover the arithmetical lessons of this lost treasure. Their efforts are often hampered, however, due to the condensed and obscure manuscripts which date back to the eleventh century. Essentially, one can only attribute the mysterious disappearance of Rithmomachia to the period in which it was lost, namely the Dark Ages.

The name of the game is of Greek origin. 'Rithmo' is derived from arithmos and rythmos. Arithmos means number, and rhythmos, besides the meaning rhythm, also pertains to the number and proportion of numbers in the Middle Ages. 'Machia' is derived from machos, which means battle. Therefore, Rithmomachia is simply the Battle of Numbers. (Mebben, 1)

The aim of each combatant is to win an honorable victory by meeting, intercepting, ambushing, and surrounding carefully constructed number pieces. It is a strategy game, similar to that of chess. Chess, however, was played as a tactical war game specifically designed for entertainment purposes. Rithmomachia's aim is not for foes to fight against one another but to take part in a contest of intellectual wits. Features of Boethius's number theory and Pythagoras's number symbolism recur in the game of Rithmomachia. Due to these components, many scholars considered this game to be a useful and enjoyable teaching aid for some of mathematics' greatest discoveries.

Many presume that the inventor of Rithmomachia is Boethius or perhaps even Pythagoras. The oldest piece of written evidence dating back to 1030, however, depicts the original creator to be a monk by the name of Asilo. It is questionable whether or not he actually invented the game or just simply elaborated on an existing game of that time period. Perhaps the true inventor still remains nameless in the midst of the Dark Ages.

Throughout history, various mathematicians and scholars including Faber, Abraham Ries, Selenus, and Boissiere have studied Rithmomachia and recorded their observations. In the eleventh and twelfth centuries, research focused on checking the existing rules and elaborating on them in order to enhance the game itself. As time progressed, these modifications were recorded and the awareness of the general public grew. Rithmomachia's popularity, however, reached its highest level at the time of book printing. Written evidence has been preserved in Latin, French, Italian, and German indicating its European dominance. Some of these texts focus on the mental recreation of the game whereas others depict Rithmomachia as a training exercise for the study of arithmetic. Public advertisements have even been uncovered in relation to the sale of the board and its pieces. Description of such sets range from a simple cloth chessboard and wooden pieces to a marble board inlaid with gold or silver and its exquisite pieces. (Thibault, 1) Unfortunately, Rithmomachia sets are no longer available to purchase so individuals must invent their own facsimile of the board game.

At the end of the seventeenth century, Rithmomachia began to dissipate into the Dark Ages. Mathematics was changing, represented by numerous factors such as the introduction of zero, calculations of fractions, and integration and differentiation of integrals. Some researchers, however, attribute Rithmomachia's disappearance to the overwhelming popularity of Chess. No schooling was required to play a game of chess, whereas players of Rithmomachia needed to be schooled in several arithmetic principles in order to be successful. Therefore, chess became the more appealing strategy game to the general public. Regardless of the reasons for its disappearance, the tragedy is that there is little archaeological evidence which support its existence. An exact description or even the general rules are very difficult to decipher from the lost remains. Due to this fact, reconstruction becomes a very tedious and challenging undertaking.

Rithmomachia is played by two people on an eight by sixteen checkered board. This is equivalent to two modern chess boards. The pieces, however, must be constructed by hand. Each player must have twenty-four pieces. Black has the odds, and White has the evens. Each side has eight circles, eight triangles, seven squares, and one pyramid. Numbers are constructed according to the number theory of Boethius. The Odd pieces all come from 3, 5, 7, and 9 whereas the Even pieces are derived from 2, 4, 6, and 8 as shown in Table 1.

Odds

Circles	3	5	7	9	x
Circles	9	25	49	81	x^2
Triangles	12	30	56	90	x(x+1)
Triangles	16	36	64	100	(x+1)^2
Squares	28	66	120	*190	(x+1)(2x+1)
Squares	49	121	225	361	(2x+1)^2

Evens

Circles	2	4	6	8	y
Circles	4	16	36	64	y^2
Triangles	6	20	42	72	y(y+1)
Triangles	9	25	49	81	(y+1)^2
Squares	15	45	*91	153	(2y+1)^2

Table 1 Rithmomachia Pieces

Those numbers marked with the asterisks are the pyramid pieces. The 190 pyramid on the Odd's side is made up of a square 64, a square 49, a triangular 36, a triangular 25, and a round 16. This is evident since $190 = 8^2 + 7^2 + 6^2 + 5^2 + 4^2$. On the Even side, the 91 pyramid is composed of six pieces instead of five. A square 36, a square 25, a triangular 16, a triangular 9, a round 4, and a round 2 are its corresponding pieces since $91 = 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2$. Essentially, the game would be more consistent if both pyramids had the same number of pieces in them but unfortunately this is not the case. The number corresponding to 190 in Table 1 is 153 which is not a sum of consecutive squares. On the other side, the number corresponding to 91 is 120. The same argument applies here as well. Evidently this inconsistency cannot be resolved.

The starting positions of the pieces at the opening of the game are shown in Figure 1. The lower set of pieces correspond to the Evens and the upper set is the Odds. Dots or other forms of marking are often placed somewhere on the edge in order to designate the bottom of each piece. Therefore, 6 would not be confused with 9 and so forth. Each player is allowed one move on his/her turn. Movement of pieces is determined by the shape and not the corresponding number. This movement is furthermore restricted to only the horizontal and vertical directions, excluding the diagonal option. Circles move one space, triangles move two, and squares and pyramids move three. Moving is contingent, however, on whether or not the intervening spaces are empty. Thus, a piece may become blocked and unable to move in any direction.

The method of fighting in Rithmomachia is determined by the choice of capture. One type and perhaps the simplest form of battle is an encounter. If a piece is so placed that in its next regular move it could take the place of an opponent's piece with the same number, then the opponent's piece is taken away. For example, the 25 triangular piece belonging to the Evens could

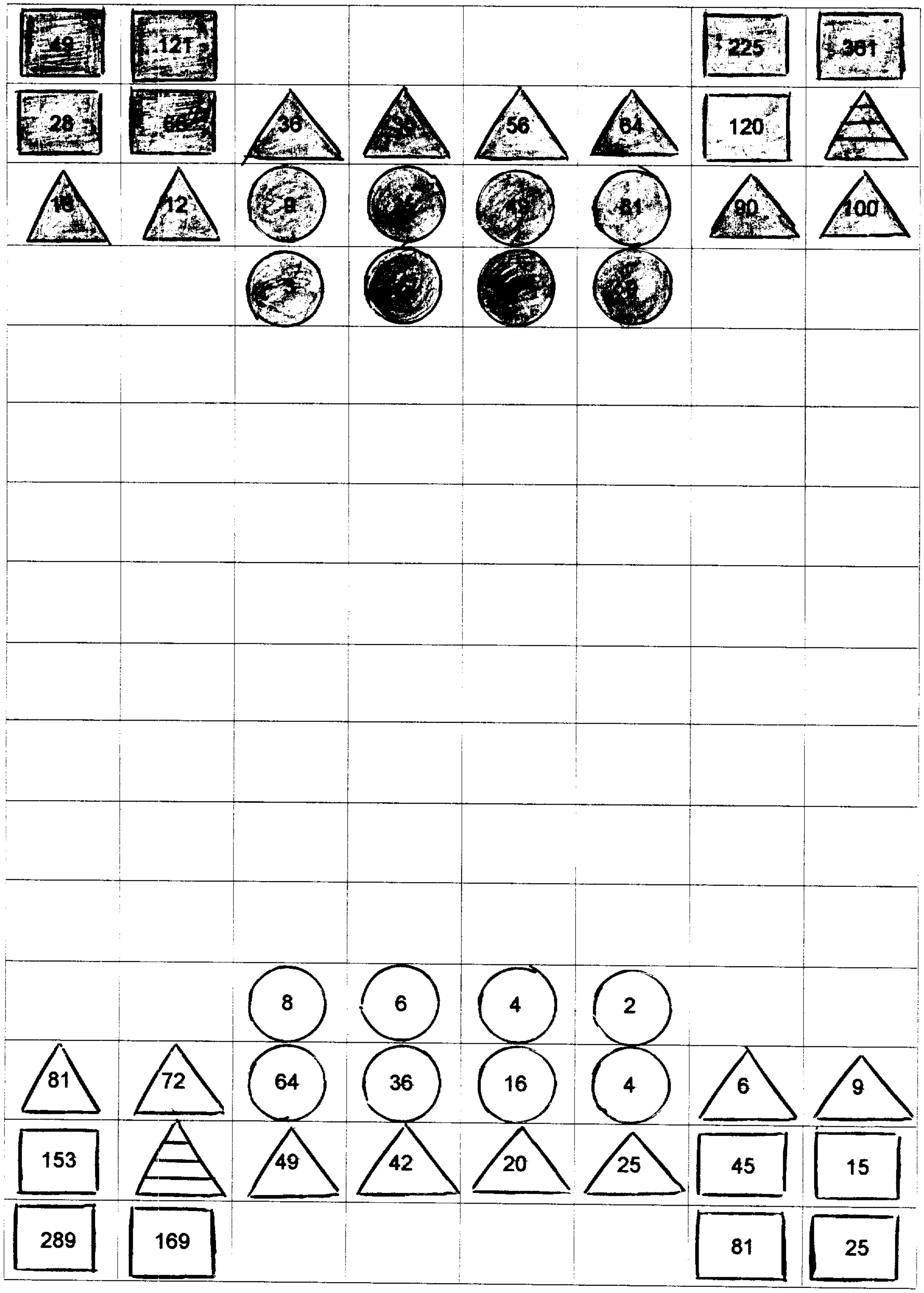


FIGURE 1 The Initial Position

capture the Odd 25 circular piece if there was one empty space between the two. On the other hand, the Odd 25 circular piece could only capture the Even 25 triangular piece if there were no empty spaces between the two. Similarly, this same argument applies to other pieces as demonstrated in Figure 2. Once an encounter has occurred, the victor occupies the fallen enemy's position and play continues. In all, there are only seven numbered pieces which reside in both the Even and Odd sides, namely 9, 16, 25, 36, 64, and 81. Therefore, there are just as many winners by encounter for Even as there are for Odd.

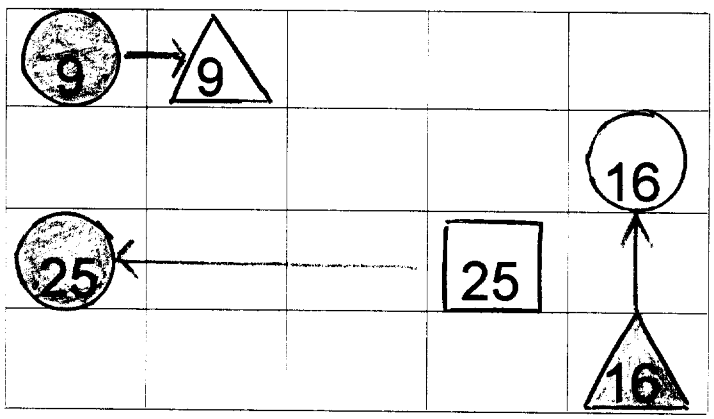


FIGURE 2 Captures By Encounter

The second choice of capture is by sally. A smaller number can attack a larger one if it becomes equal to it when multiplied by the spaces lying between it and the larger number. It is important, however, that the intervening spaces are empty. Another consideration is that pieces must move in a straight line in order to claim the opponent's position on the board. For example, circle 9 can capture square 81 if there are nine empty spaces between them. Other examples are presented in Figure 3. The triangle 30, however, could never be captured from circle 2 since the board is not long enough for fifteen spaces to lie between them. Thus, some pieces are forever safe from being captured by sally. Table 2 shows possible combinations which satisfy the necessary conditions. After analyzing all the possible combinations, one can see a distinct advantage to having Evens when comparing the number of captures by sally.

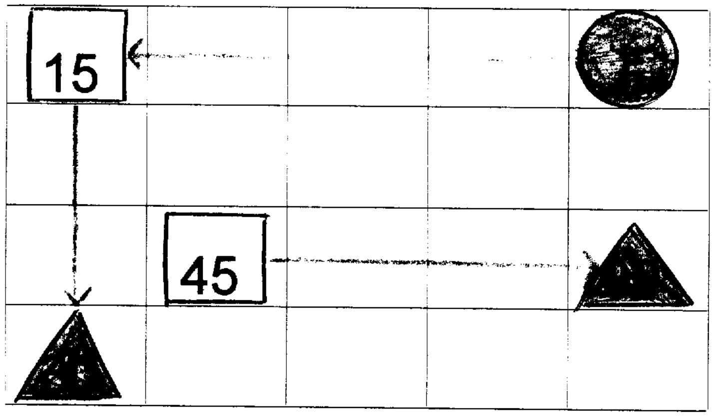


FIGURE 3 Captures By Sally

Evens Against Odds

Even pieces when separated by the appropriate number of empty spaces can capture Odd pieces by sally.

Attacking Even Piece	Empty Spaces	Captured Odd Piece
2	6	12
4	3	12
2	8	16
4	4	16
8	2	16
4	7	28
15	2	30
4	9	36
9	4	36
4	14	56
8	7	56
8	8	64
16	4	64
9	9	81
9	10	90
15	6	90
45	2	90
20	5	100
20	6	120
25	9	225

Odds Against Evens

Odd pieces when separated by the appropriate number of empty spaces can capture Even pieces by sally.

Attacking Odd Piece	Empty Spaces	Captured Even Piece
3	2	6
3	3	9
3	5	15
5	3	15
5	4	20
5	5	25
9	4	36
12	3	36
3	12	36
3	14	42
7	6	42
5	9	45
7	7	49
16	4	64
36	2	72
12	6	72
9	9	81
7	13	91

TABLE 2 Captures By Sally

Two smaller numbers can ambush a larger one, if they are placed that their united strength is the same value of the larger enemy. These two pieces, however, must be played by legal moves to the square occupied by the enemy piece and their sum must equal its value. Examples are given in Figure 4. Variations of the game have allowed for players to use multiplication or division instead of addition or to use more than two pieces in an attack. Once a player experiments with the initial meaning of an ambush, then these modifications can be made in order to stimulate new avenues of thought and mastery.

Once an ambush has been achieved, either of the two pieces can replace the captured opponent's square. For example, 4 and 8 from the Evens can capture 12 on the Odd's side and either one of them can claim the enemy's position on the board. Table 3 indicates other captures by ambush. Once again, there is a distinct advantage to possessing the Even pieces since there are more possible ambushes than what is allowed on the Odd's side.

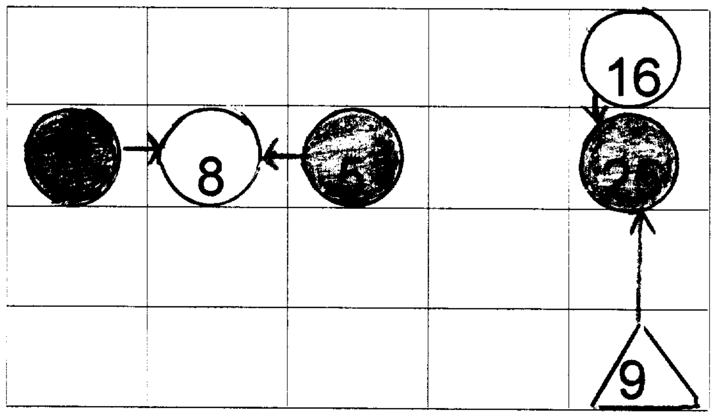


FIGURE 4 Captures By Ambush

Evens Against Odds

Evens To Be Added Together		Result
4	8	12
8	20	28
16	20	36
4	45	49
15	49	64
2	64	66
9	72	81
36	45	81
9	81	90
9	91	100
49	72	121
72	153	225
72	289	361

Odds Against Evens

Odds To Be Added Together		Result
3	5	8
3	12	15
7	9	16
9	16	25
12	30	42
9	36	45
28	36	64
16	56	72
25	56	81
25	66	91
49	120	169
64	225	289

TABLE 3 Captures By Ambush

The siege is the last form of capture. When a piece is surrounded in such a manner that they cannot save themselves either by flight or by the help of their companions, then they are forced to surrender. In contrast to encounters, sallies, and ambushes, the victor does not assume the fallen opponents' square. Another difference is that any number is subject to capture by a siege. There are, however, certain pieces that can only be taken by a siege; these are listed in the following table.

Evens	Odds
2	3
4	5
4	7
8	190

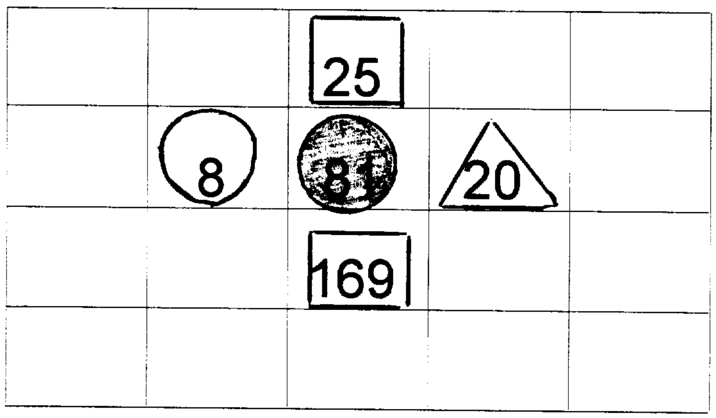


FIGURE 5 Capture By Siege

Pyramids are unique in that they can be captured as a whole entity or broken down into single components and then be attacked. When trying to capture a pyramid whole, it is important to note that 190 can only be taken by siege. The 91 pyramid, however, can be attacked through sally, ambush, or siege since 7 times 13 equals 91 and 25 plus 66 yields the same result. Therefore, the Even pyramid is at a distinct disadvantage in comparison to the

Odd's side. Recalling from previous discussion, it was noted that the Even pyramid is made up of six pieces whereas the Odd's pyramid has only five. This is significant since attacks can also be made by any of the four described methods in relation to the layers of a pyramid. Thus, a balance has seemed to emerge between the two opponents' pyramid pieces.

Attacks in relation to a single layer must be made to the piece currently residing on the bottom of the pyramid. So, 64 would be the first component to attack for the Odd's pyramid and similarly 36 would be for the Even side. If single components have been removed from previous attacks, it is possible to capture the pyramid by its new total sum. Be advised, however, that pyramids may not capture enemy pieces with their new value.

There are two kinds of victories categorized as common or proper which can be achieved in this intellectual contest. The common victories are listed as the victory of the body, goods, law-suit, honor, and a combination of law-suit and honor. Proper victories, on the other hand, are designated the great, the greater, and the excellent. As ability level increases so should the choice of victory. In other words, beginners should concentrate on the common victories before progressing to the proper ones.

The simplest victory is the victory of the body. The first player to capture the number of pieces agreed upon at the start of the contest is indeed the winner. No attention should be placed on the value of the digits which reside on the captured pieces. Boissiere offered four as one choice for the definite number of pieces. As familiarity of moves and patterns of play increase, this number may grow in order to extend the game and create new avenues of strategic planning.

The victory of goods is attained by the first player to capture pieces whose total is a number agreed upon at the start of the game. Research, however, is not conclusive in regards to whether this sum must have been arrived at exactly or would be exceeded. The number of pieces does not matter in this form of conquest. If the agreed upon value is 100, a winner may be decreed if 64 and 36 are captured or if the sole piece 100 is attained. Obviously, the game would be much more challenging if the agreed upon sum is to be arrived at exactly so keep this in mind when embarking on your own Battle of Numbers.

The difficulty level increases as a player progresses to the victory of the law-suit. The object of this game is to equal or exceed a fixed total by capturing a specified number of pieces. Therefore, there are two constants which must be determined before play begins. For example, victory could be attained by the first player to equal 100 with 8 pieces. Pieces numbered 3, 5, 7, 9, 9, 12, 25, and 30 are one example of a successful assemblage of pieces. A winner, however, is not declared until all eight pieces have been captured. Even though the sum of a collection of seven pieces may exceed 100, there is still no victory of law-suit to be claimed.

The victory of honor is quite similar to that which is described by the victory of law-suit. The only difference is that players are not allowed to exceed the agreed upon sum of the captured pieces. If 100 is the target goal with only 8 pieces, then a combined total of 101 would not achieve the victory of honor. Therefore, it is possible for a player to eliminate their chances of success simply by capturing the wrong piece. In this example, a player who captures the 120 triangular piece would be eliminated from competition.

The last common victory is a combination of the law-suit and honor components. The combined total, the number of pieces, and the number of digits on the captured pieces are all fixed before play commences. This form of victory obviously restricts a player to a specific list of captures and increases the difficulty immensely. To win a game of 160 total, 5 pieces, and 9 digits, an acceptable collection of pieces would be 5, 25, 30, 36, and 64. Players at this level must be skilled in the strategic engineering of pieces in order to achieve victory. A beginner would not fare well here.

In order to be classified as a proper victory, pieces must be arranged in the enemy's line so that they are united in a certain proportion. The game concludes when one player has built a harmony in the opponent's field. Pieces regardless of color must therefore be arranged in a straight line equidistant from each other. A great victory is reached by an arithmetical, geometrical, or musical harmony of three pieces. Examples of each are given in Table 4. In regards to the first case, the difference between the two smaller numbers must equal the difference between the two larger numbers. *[(b-a) = (c- b)]* The pieces 2, 4 and 6 satisfy this criteria since there is a difference of two between each set of numbers. For a geometrical harmony, the ratio between the two smaller numbers must equal the ratio between the two larger numbers. *[(a/b) = (b/c)]* Given the pieces 2, 4, and 8, the corresponding ratio is derived as one-half. Thus, these pieces could most certainly be used to create a geometrical harmony. In the final case, the ratio of the smallest and largest numbers must equal the ratio between the difference of the two smaller numbers and of the two larger numbers. *[(a/c) = (b-a)/(c-b)]* A musical harmony could, therefore, be developed from the numbers 6, 8, and 12. After calculating the appropriate ratio which in this case is one-half, the musical harmony would then be declared. Utilizing tables of various harmonies would most certainly aid the players in their quest to achieve a great victory.

Arithmetical:	2	3	4
	3	5	7
	4	8	12
	5	25	45
	6	36	66
	7	64	121
	9	81	153
	15	120	225
	42	81	120

Geometrical:	2	4	8
	4	6	9
	9	15	25
	16	36	81
	36	42	49
	49	91	169
	64	120	225
	100	190	361

Musical:	2	3	6
	3	5	15
	5	9	45
	8	15	120
	25	45	225
	72	90	20

TABLE 4 Some Possible Harmonies for a Great Victory

The greater victory is gained by building two different types of harmonies while utilizing four separate pieces. Once again, these pieces must be arranged in the enemy's lines as described previously. The set of four pieces 9, 81, 153, and 289 fit this criteria since it contains the arithmetical progression 9, 81, 153 and the geometrical progression 81, 153, 289. Other collection of sets are denoted in Table 5.

Arithmetical/Musical				Geometrical/Musical				Arithmetical/Geometrical			
3	4	5	6	2	3	6	12	2	3	4	8
3	5	7	15	3	4	6	12	2	4	8	12
5	7	9	45	4	6	12	36	3	9	15	25
6	7	8	12	5	9	45	225	4	8	16	28
9	12	15	45	9	12	16	72	5	9	15	25
12	15	20	28	9	25	45	225	9	45	81	225
15	30	36	45	20	30	36	45	16	20	25	30
30	36	42	45	25	30	36	45	36	42	49	56
72	81	90	120	25	45	81	225	72	81	90	100

TABLE 5 Some Possible Harmonies for a Greater Victory

The final proper victory is entitled excellent since the mastery of uniting pieces at this level is indeed remarkable. In order to claim victory, a collection of four pieces must be assembled so that all three harmonies are present. Thus, a set of four numbers must contain an arithmetic, geometrical, and musical progression. Due to the complexity of this victory, there are a limited choice of sets which will achieve this desired outcome. The following table consists of eight distinct sets which will indeed claim an excellent victory.

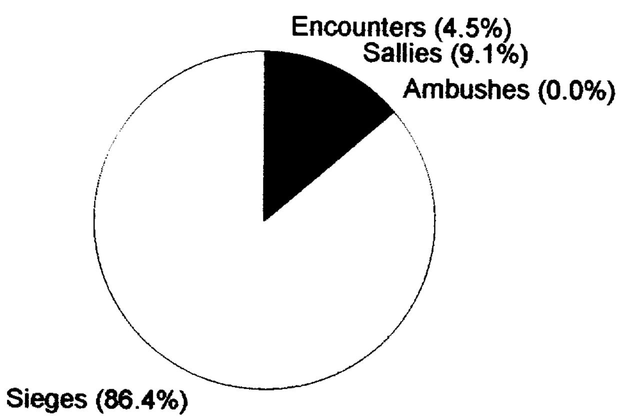
2	3	4	6
3	5	15	25
4	6	8	12
4	6	9	12
5	9	45	81
5	25	45	225
6	8	9	12
12	15	16	20

In order to see the beauty of Rithmomachia, I decided to conduct several experiments in which individuals of various age levels could take part in this intellectual contest. The first series of games were conducted with the seventh grade advanced math class at Queen of the Rosary School in Elk Grove Village, Illinois. There were ten boys in attendance and six girls. The second session consisted of thirteen eighth grade algebra students at the same school. This group consisted of five boys and eight girls. Groups of two or three players were randomly generated based on arrival time. The experiment began with a short introduction pertaining to the general rules of play. Throughout the session, the total number of captures, encounters, sallies, ambushes, and sieges were recorded for each of the individual games. These numbers can be compared using the following table. Groups 1-3 correspond to the seventh grade participants and groups 4-6 were composed of eighth grade students.

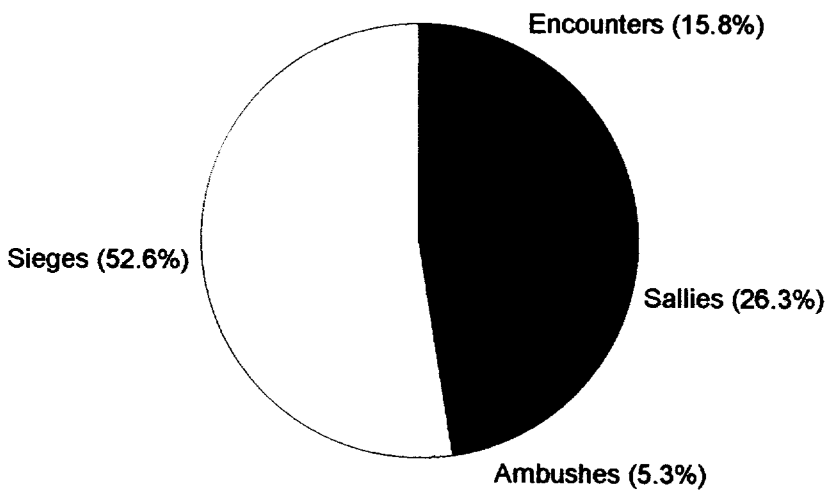
	Total Captures	Encounters	Sallies	Ambushes	Sieges
Group 1	22	1	2	0	19
Group 2	19	3	5	1	10
Group 3	10	0	4	0	6
Group 4	15	1	7	0	7
Group 5	9	1	4	0	4
Group 6	7	1	1	1	4

After about an hour of exploration, general observations were made as a class. Topics of discussion included the desired choice of capture, attempted strategies, and other general comments pertaining to the game. It is important to note, however, that pyramids were not used in either of the two middle school sessions. This decision was made in order to alleviate some of the complexity of the game. The following pie charts offer a look at each of the individual games as well as a summative look at the two distinct grade levels.

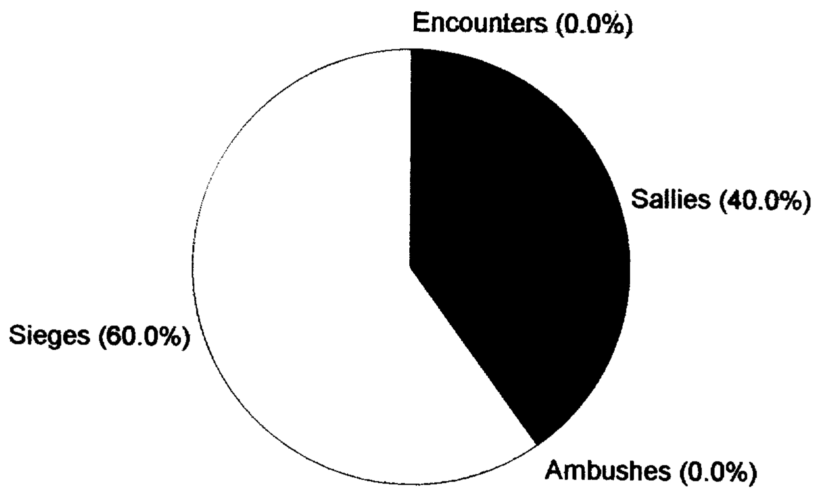
Group 1 Observations



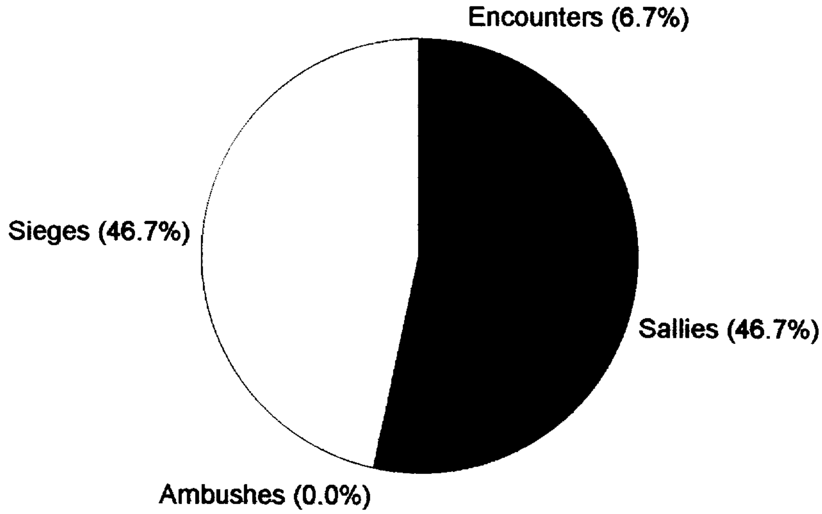
Group 2 Observations



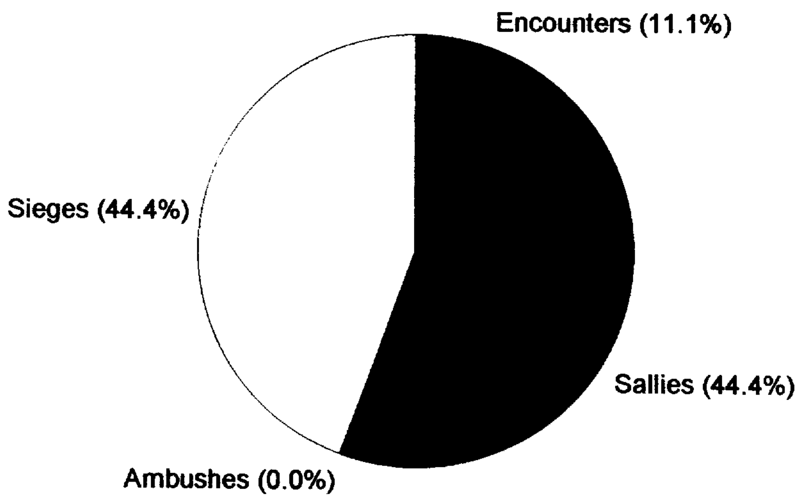
Group 3 Observations



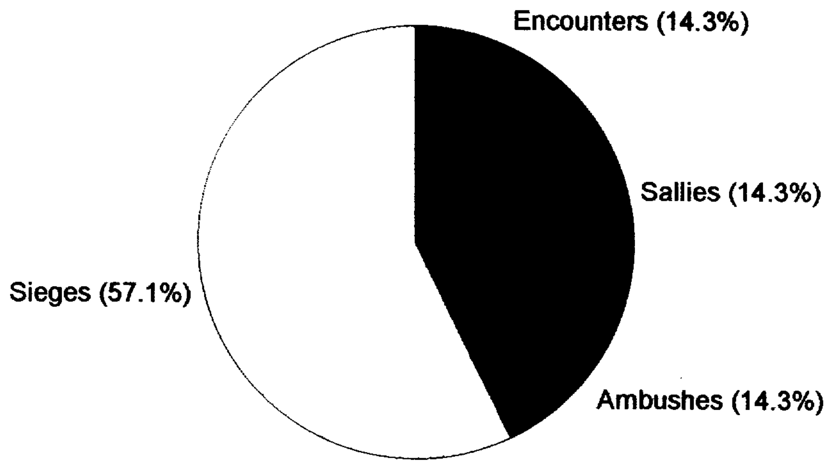
Group 4 Observations



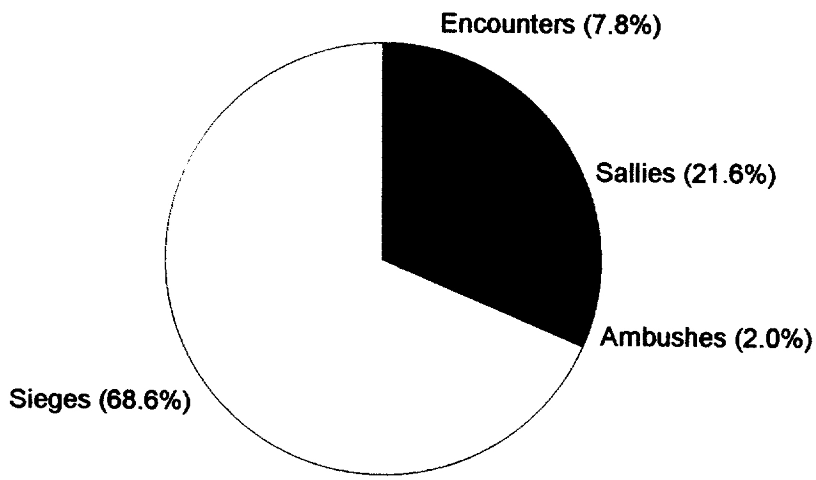
Group 5 Observations



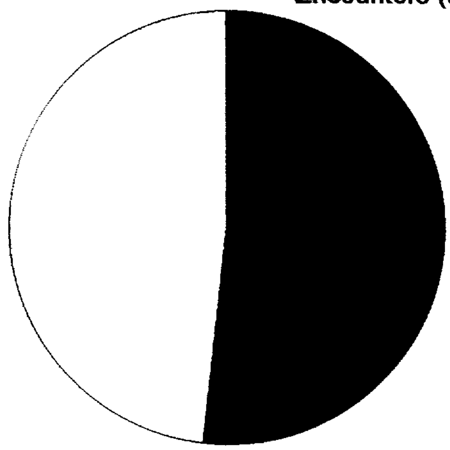
Group 6 Observations



Seventh Grade Rithmomachia Observations



Eighth Grade Rithmomachia Observations



Predominant choice of capture for the seventh grade students was the siege. The boys, in particular, felt the need to attack and move into the opponent's playing field. Many times this resulted in sacrificing one's own pieces. This strategy is evident upon viewing the previous pie charts specifically related to the seventh grade participants. For Group 1, siege was the most popular choice of capture, and their combined total was the largest out of all three groups. Thus, an aggressive approach yields a greater number of captured pieces for both sides.

Group 2, on the other hand, utilized a variety of captures. In fact, they successfully completed each of the four described methods. Their focus was on using different forms of captures in order to surprise their opponent. By not concentrating on one specific piece and setting up a variety of attacks, often motives were disguised and therefore unrecognizable.

Group 3 seemed to keep the strategy of using a defensive and at the same time offensive approach. One student even spoke of the connection of Rithmomachia to a football game. Creating offensive and defensive pieces, in his opinion, was the key to victory. Pieces of smaller value and typically circular were used as the running backs or wide receivers. Larger pieces of triangular or square shape served as the center or defense tacklers. To me, this analogy was not only a creative strategy but an effective one as well.

Being a year older and perhaps brimming with maturity, the eighth graders seemed to take a more scientific approach to the problem at hand. At the start of the game, Groups 4 and 5 attacked only by sally. In fact, many students were furiously punching away numbers on their calculator in order to not only devise attacks on their opponent's pieces but to protect their own as well. At this point another observation was made. Some pieces could never be captured by sally, and therefore a new strategy must be developed. Thus, other forms of capture were then attempted through careful thought and planning.

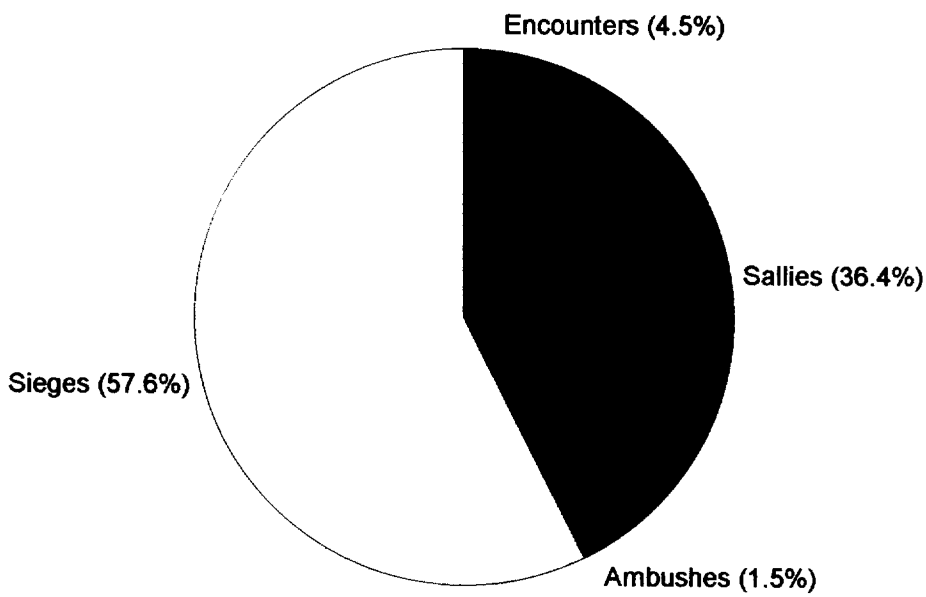
Each group definitely had a designed offensive and defensive strategy in action. Group 4 assigned roles to each of their team members. One person would focus on defending pieces while another would concentrate on the attack. It was rather amusing since each team would draw diagrams in their notebooks and hide them so the opposing eyes could not see. While working collectively, their game appeared to not only be exciting but competitive as well. Groups 5 and 6 were very timid at first. Neither group seemed to want to venture into the opposing field of play. After the discussion component of the experiment, these groups began to assume the same strategy as Group 4. At this point, they seemed to be capturing pieces more frequently while utilizing a variety of strategic attacks.

These games were not played in accordance with any of the previously described victories. Individuals groups, however, deemed the winner as the one with the most pieces at the end of the two hours. In my opinion, they were all winners. This middle school experience was indeed special since the energy and enthusiasm of these students fueled my own. It is an experience that I will most certainly replicate in my own classroom.

The second component of this experiment consisted of several independent games where players ranged from twenty to fifty years old. Time allotted for these demonstrations varied depending on the situation. The total number of captures, encounters, sallies, ambushes, and sieges were also recorded. These numbers can be observed in the following table and pie chart.

	Total Captures	Encounters	Sallies	Ambushes	Sieges
Group 7	21	0	6	0	15
Group 8	13	0	2	1	10
Group 9	5	1	3	0	1
Group 10	6	1	4	0	1
Group 11	10	1	5	0	4
Group 12	11	0	4	0	7

Adult Rithmomachia Observations

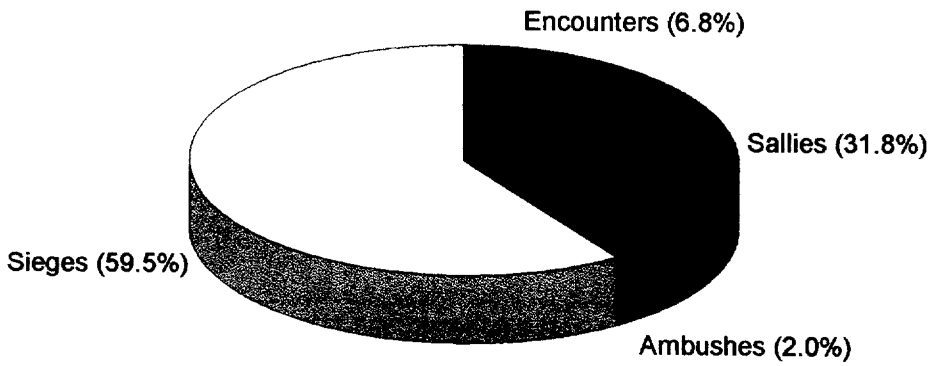


In Games 7-12, the favorable captures were achieved through sally and siege. One of the more interesting strategies demonstrated in Game 7 was to push the opponent back into a corner. Thus, multiple pieces could not escape, and each of them were eventually captured by siege. Circular pieces are often able to escape a siege due to their increased mobility, but backing them into one of the four corners seemed to be an effective method of securing their capture.

Many of these adult players continued to ask if there were other ways of securing an attack. Operations such as division or subtraction were often suggested. This statement most certainly supports the numerous adaptations which were made to the game before its disappearance. Implementing these types of operations not only increase the difficulty of the game, but they also spark new avenues of strategic play. If I were to expand upon this study, I would most certainly introduce some of these concepts in order to delve deeper into the world of Rithmomachia.

Pyramids were used in these adult experiments unlike those conducted at the middle school level. In most cases, they remained in their original spot on the board. Also, there were no attacks made in any game on the individual layers of a pyramid. There were two instances, however, in which a pyramid was captured as a whole entity. The White pyramid 91 was captured by siege in Games 7 and 8. The Black pyramid, however, remained unchallenged. Due to the limited amount of data in relation to pyramid play, it is still uncertain whether or not there is an advantage to possessing either one of these pieces.

In all, there were 148 captures throughout these twelve games. The percentage of captures for each particular type are denoted in the following pie chart. As you can see, siege was the most popular and successful choice of attack. Sallies, encounters, and ambushes followed respectively in that order. This corresponds to the given research since any piece can be captured by siege. Also, by comparing the lists for ambushes and sallies, it is obvious that sallies are more attainable, and therefore the corresponding percentage should be higher. Given the fact that this was the first Rithmomachia experience for any player, it is expected that mastery would not be achieved in regards to each of the described forms of capture. Since a limited number of pieces can be captured by ambush and encounter, these forms of capture were often forgotten. This observation is reflected once again in the following pie chart.



While observing the collected data, it is also important to note the shape and designated value of each captured piece. The following table tallies the number of times a particular piece was captured throughout all twelve games. Notice that some pieces are not listed, and therefore were never captured in any of the preceding experiments.

Odd Captured Pieces

Shape	Value	Tally
Circular	3	2
Circular	5	4
Circular	7	2
Circular	9	7
Triangular	12	10
Triangular	16	9
Circular	25	1
Square	28	3
Triangular	30	2
Triangular	36	3
Circular	49	1
Square	49	3
Triangular	56	2
Triangular	64	2
Square	66	4
Circular	81	6
Triangular	90	5
Triangular	100	2
Square	120	2
Square	121	1
Square	225	2
Square	361	2
	Total	75

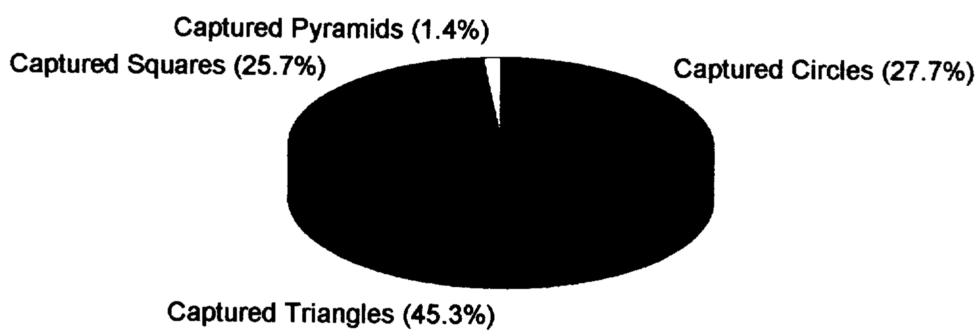
Even Captured Pieces

Shape	Value	Tally
Circular	2	2
Circular	4	3
Triangular	6	5
Circular	6	6
Circular	8	4
Triangular	9	9
Square	15	5
Triangular	20	2
Triangular	25	4
Square	25	2
Circular	36	3
Square	45	7
Triangular	49	1
Triangular	72	7
Square	81	1
Triangular	81	4
Pyramid	91	2
Square	153	4
Square	169	1
Square	289	1
	Total	73

The shape of a piece is critical when trying to maneuver pieces in order to set up attacks. Group observations concluded that circles were the most difficult pieces to capture by siege due to their increased mobility. Another factor is that the value of many of the circular pieces is quite low. Therefore, these numbers are unable to be attacked by sally or ambush. The role of the circular piece, in my eyes, is the attacker. Losing a multitude of these pieces would most certainly decrease a player's chance of claiming victory.

Square pieces, on the other hand, seemed relatively easy to capture by siege since their movement is quite restricted. Players did conclude, however, that the difficulty level increased when attempting sallies or ambushes on these particular pieces. Many times attacking pieces could not be placed in the appropriate position on the board in order to claim victory, and therefore the square piece was saved. Another factor which supports the survival of these shaped pieces resulted specifically from the nature of the experiment. Due to limited time of play, many of the square pieces remained in their original starting positions on the board and hence were never in position to be captured.

Most players agreed that the triangular pieces were the easiest to capture. The movement of these pieces are limited to every two spaces on the board. Therefore, sieges can be attained rather quickly. In addition, the value of most triangular pieces are convenient for divisibility purposes. Thus, sallies become a popular choice of capture. Since sallies and sieges were the predominant choice of capture for the experiment, the collected data does mirror these noted observations. The following pie chart is yet another comparison tool which depicts the percentages of captured pieces for each given shape. As discussed previously, triangular pieces did emerge as the most popular type of captured piece.



Throughout these twelve games, 75 Odd pieces were captured and 73 Even pieces were captured. Based on this analysis, the Even and Odd sides do appear to be relatively balanced. In the individual contests, there were more Odd pieces captured in 6 of the experiments deeming the Even side as the victor. Five of the games consisted of more Even captured pieces claiming the Odd side as the winner. Thus, one game ended in a tie. Based on the research, I was expecting the Even side to dominate in the majority of contests. Due to the inexperience of players and the evident time constraints, this analysis was not directly observed. In fact, many of the adult players felt that the advantage lie in the Odd side instead. Perhaps with more time and a specific victory in mind, the advantage of possessing the Even side would be apparent. Another consideration is that these experiments involved the four most basic forms of capture. Modifications such as using more than two pieces for an ambush or the implementation of different operations would most certainly be other factors to consider.

Throughout history, the emergence and disappearance of games can be directly seen regardless of the given time period. Canasta is just one example of a game which has also passed into oblivion. Fashion changes and this fact is not only reflected in the clothing industry but in the entertainment world as well. Perhaps the revival of this game will be seen in my lifetime. More than likely, however, it will remain buried in the midst of the Dark Ages. The people who played the game have long been forgotten and the challenge and mystery which surrounds Rithmomachia is buried with them. In order to experience the rebirth of this lost treasure, the excitement which once surrounded its existence must also emerge. The intense power of numbers to fascinate the human mind can indeed bring a breath of fresh air into this lifeless beauty of Rithmomachia. The urgency to discard its relevance in today's society is the only signal we need to determine that the opportunity to excel and learn is at hand. Learning from the past is inevitable but passing up on its lessons is unfathomable.

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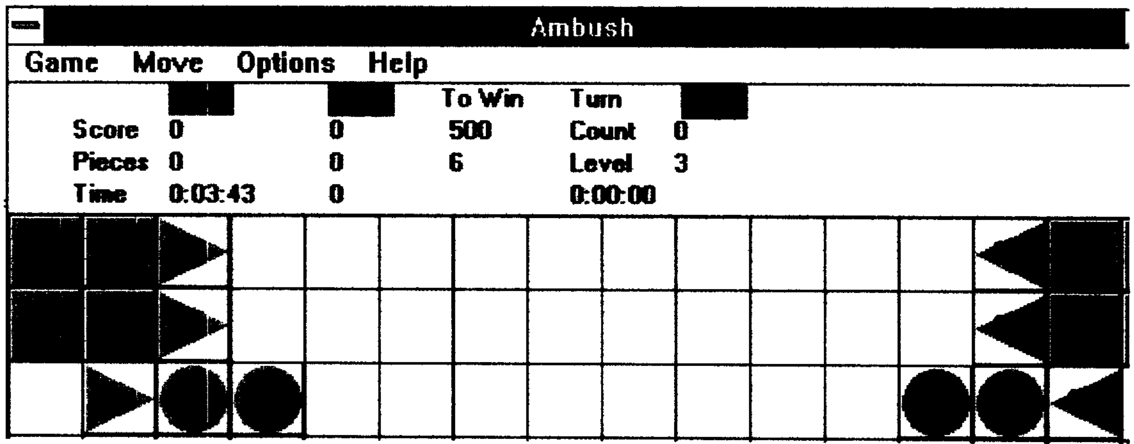
*ShareWare offers a software package which allows a user to play Ambush, which is similar to Rithmomachia. The price is $19.99 + $3.00 Shipping and Handling. The free version has limited levels and no person vs. person mode.

Ambush



Actual board is 16 by 8

- Multiple difficulty levels
- Play against a friend. Practice with the computer.
- See what the computer is thinking
- Force the computer to make the best move calculated so far
- Save and Restore games
- Hints
- At least a 486 recommended
- **Ambush** is carried on many Shareware CDs

- **Ambush** is an ancient board game somewhat reminiscent of chess. At one time it was very popular but it is little known today. In the literature on board games it is called Rithmomachia (you can see why we called it **Ambush**). There are 3 basic pieces : rounds, which move 1 space at a time, triangles that move 3 spaces, and squares which move 4 spaces. There is also a pyramid which is composed of each the other piece types and can move like any of them. Each Piece is assigned a numerical value and movement is in any direction. Captures are accomplished by various spatial and numerical combinations. Price: $19.99 + $3.00 Shipping and Handling.
- Requires vbrun200.dll in your windows\system directory and pkunzip.exe.

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RITHMOMACHIA --A GAME FOR MAGES?
by Daniel U. Thibault

(Based on an article by Michel Boutin in Jeux & Strategie #26, April-May 1984; the translation is mine)

GURPS applicability

Recommended for a Fantasy setting, this game would be played by the Mages and other members of the intellectual elite. It is deeply steeped into numerological mysticism, and might even be used for divinatory purposes. It would be perfectly unintelligible to the non-initiated, and thus presents quite a few interesting role-playing possibilities!

Rithmomachia: A Hobby skill, Mental/Hard (or Very Hard), no default, prerequisites Literacy and Mathematics (at IQ). Mathematical Ability helps.

A game set varies from the simplest (pouch of rough wooden pieces and a cloth chessboard; $20, 1 lb) to the exquisite (pieces and board of marble or precious wood inlaid with gold and silver; $1,000 and up, 10-20 lbs).

Historical Background

This complex chess-like game appeared in the western world around the year 1000. The game knew a great burst of popularity in the XVth century, because of some rules changes. When chess also saw its rules change (particularly when the Queen started to move in its modern fashion instead of its previous King-like motion), Rithmomachia started fading rapidly, at the close of the XVIth century. The rules given here are those established in 1556 by Claude de Boissiere, a Frenchman.

Introduction

Rithmomachia is latin for "Battle of Numbers"; the game is at once a battle of pawns and a battle of numbers. To play, one must be able to do a lot of quick mental arithmetic, as we shall see. This is why this game was, throughout its period, very elitist, being played mostly by high ranking churchmen and nobles. The game is flawed, in that the two sides are not equal (though you'll probably find as many experts swearing the white pieces are the better ones than you'll find opting for the black pieces!).

Components

- A 16 by 8 chessboard

- 57 pawns, 29 black ones and 28 white ones, broken down as follows:

White pieces

- 8 round pieces, of values
  3 / 5 / 7 / 9 / 9 / 25 / 49 / 81
  (odd numbers 3 to 9 and their squares)
- 8 triangular pieces, of values

```
    12 / 30 / 56 / 90 / 16 / 36 / 64 / 100
    (3*4 / 5*6 / 7*8 / 9*10 and the squares of 4 / 6 / 8 / 10)
  - 7 square pieces, of values
    28 / 66 / 120 / [190] / 49 / 121 / 225 / 361
    (4*7 / 6*11 / 8*15 / [10*19] and the squares of 7 / 11 / 15 / 19)
  - the preceding 190 piece is broken down into a pyramid of
    round      16 (4*4)
    triangular 25 (5*5)
    triangular 36 (6*6)
    square     49 (7*7)
    square     64 (8*8)
```

A pyramid is made up of stacked pieces. The pawns destined to be stacked are of progressivly smaller sizes, the larger value at the bottom, the smaller on top.

Black pieces

```
  - 8 round pieces, of values
    2 / 4 / 6 / 8 / 4 / 16 / 36 / 64
    (even numbers 2 to 8 and their squares)
  - 8 triangular pieces, of values
    6 / 20 / 42 / 72 / 9 / 25 / 49 / 81
    (2*3 / 4*5 / 6*7 / 8*9 and the squares of 3 / 5 / 7 / 9)
  - 7 square pieces, of values
    15 / 45 / 153 / 25 / 81 / 169 / 289
    (3*5 / 5*9 / [7*13] / 9*17 and the squares of 5 / 9 / 13 / 17)
    (note that 3+2=5 / 5+4=9 / 7+6=13 / 9+8=17)
  - the preceding 91 piece is broken down into a pyramid of
    round       1 (1*1)
    round       4 (2*2)
    triangular  9 (3*3)
    triangular 16 (4*4)
    square     25 (5*5)
    square     36 (6*6)
```

As one can see, the game is a numerologist's dream! This went very well with the mystic of numbers that pervaded the Middle Ages. On with the rules!

All the pawns are reversible, being white on one side and black on the other. This is because they can be captured and flipped as the game progresses.

Set-up
======

```
   BLACK SIDE
 -------------------------------
| 25  81                 169 289|
| 15  45  25  20  42  49  91 153|
|  9   6   4  16  36  64  72  81|
|          2   4   6   8        |
|                               |
|   ...   six more rows    ...  |
|                               |
|          9   7   5   3        |
|100  90  81  49  25   9  12  16|
|190 120  64  56  30  36  66  28|
|361 225                 121  49|
 -------------------------------
   WHITE SIDE
```

Victory conditions
==================

Both players must agree, when starting a game of Rithmomachia, on which victory conditions to use. There are:
Seven common victories, and
Seven proper victories.

The Common Victories are:

```
The Victory of body          To take a set number of pawns
The Victory of assets        To take a set total value in pawns
The Victory of proceeds      To take a set number of digits
The Victory of body and assets
The Victory of body and proceeds
The Victory of assets and proceeds
The Victory of body, assets, and proceeds
```

The Proper Victories are broken down into:

Mediocre Victories
Great Victories
Excellent Victory

In all Proper Victories, the object is to take the opposing pyramid, and then position three or four pawns, in line or square formation, in the opponent's half of the board, so as to form an arithmetic, geometric, or harmonic progression.

In an Arithmetic Progression, the differences between successive numbers are given by a single value (called the ratio of the progression). For example: 2 - 5 - 8 - 11 is an Arithmetic Progression of ratio 3.

In a Geometric Progression, the ratios between successive numbers are given by a single value (called the ratio of the progression). For example: 3 - 12 - 48 is a Geometric Progression of ratio 4.

In an Harmonic Progression, the ratio of two successive differences is equal to the ratio of the end numbers. If the Progression is a - b - c, we have c/a = (c-b)/(b-a). This number is the progression's ratio. For example: 4 - 6 - 12 is an Harmonic Progression of ratio 3, as 12/4 = 3 and (12-6)/(6-4) = 6/2 = 3.

The Mediocre Victories are achieved by obtaining one of the progressions.

The Great Victories are achieved by obtaining two progressions at once.

The Excellent Victory is achieved by obtaining all three progressions at once.

Examples:

```
 _______________
|_16|___|___|___|
|___|_36|___|___|   This is a Mediocre Victory by
|___|___|___|___|   Arithmetic Progression (of ratio 20)
|___|___|___|_56|

 _______________
|___|___|___|___|
|__4|__6|_12|_36|   This is a Great Victory by
|___|___|___|___|   Geometric Progression (4 - 12 - 36, ratio 3) and
```

```
|___|___|___|___|  Harmonic Progression (4 - 6 - 12)

 _______________
|_12|___|__4|___|  This is an Excellent Victory:
|___|___|___|___|  Arithmetic (6 - 9 - 12, ratio 3)
|__9|___|__6|___|  Geometric (4 - 6 - 9, ratio 3/2) and
|___|___|___|___|  Harmonic (4 - 6 - 12, ratio 3)
```

Note that in all these examples, the victories were achieved with captured enemy pawns.

Movement
========

There are two types of moves: Regular and Irregular. In a Regular move, the piece slides from its starting point to its end point; the intervening squares must be unobstructed. In an Irregular move, the piece jumps from its starting point to its end point, regardless of obstacles.

Round piece movement template (* Regular, + Irregular)

```
 ___________________________
|___|___|___|___|___|___|___|
|___|___|___|___|___|___|___|
|___|___|_*_|___|_*_|___|___|
|___|___|___|_X_|___|___|___|
|___|___|_*_|___|_*_|___|___|
|___|___|___|___|___|___|___|
|___|___|___|___|___|___|___|
```

Triangular piece movement template

```
 ___________________________
|___|___|___|___|___|___|___|
|___|___|_+_|_*_|_+_|___|___|
|___|_+_|___|___|___|_+_|___|
|___|_*_|___|_X_|___|_*_|___|
|___|_+_|___|___|___|_+_|___|
|___|___|_+_|_*_|_+_|___|___|
|___|___|___|___|___|___|___|
```

Square piece movement template

```
 ___________________________
|___|___|_+_|_*_|_+_|___|___|
|___|___|___|___|___|___|___|
|_+_|___|___|___|___|___|_+_|
|_*_|___|___|_X_|___|___|_*_|
|_+_|___|___|___|___|___|_+_|
|___|___|___|___|___|___|___|
|___|___|_+_|_*_|_+_|___|___|
```

Pyramids can move like round, triangular, or square pieces as long as they still have a representative of that shape in their stack.

Captures
========

Captures must precede or follow a Regular move. When the capture precedes the move, the capturing pawn takes the captured pawn's place. When the capture follows the move, the captured pawn's place is left empty. It is possible to achieve several captures at once, both before and after the move! When capturing several pawns before a move, the capturing pawn chooses which captured pawn's place to take. Captures are NOT mandatory.

Captured pawns are flipped and may be re-introduced on the board, on any free square of the player's board edge. Putting a captured pawn down replaces a move. It is possible to capture in this way.

There are six ways to capture:

```
    The Encounter         The capturing pawn comes within one Regular
                          move of the victim.
      A   B   C   D
     _______________
  1 |___|_25|___|___|   B1: Round white      D3: Square black
  2 |___|___|___|___|
  3 |___|___|___|_25|   If white moves his piece to C2, he could
  4 |___|___|___|___|   capture black.

    The Ambush            When a number is equal to the sum,
                          difference, product, or ratio of two
opposing pawns, it can be taken on condition that both capturing
pawns be within a Regular move of the victim.
      A   B   C   D
     _______________
  1 |___|__8|___|___|   B1, A4: Round black     C2: Triangular white
  2 |___|___|_12|___|
  3 |___|___|___|___|   If black moves his 4 to B3, he can take the
  4 |__4|___|___|___|   12 as 8+4=12 and both his pieces are within
                        a Regular move of the white piece.

    The Assault           A number encounters an opposing pawn in the
                          same row, column, or diagonal so that the
number of intervening squares is equal to their product or ratio.
The intervening squares must be unoccupied.
      A   B   C   D
     _______________
  1 |___|___|___|_12|   D1: Tri. white  B3: Round black  A4: Tri. black
  2 |___|___|___|___|
  3 |___|__2|___|___|   By moving his 2 out of the way (to A2 or
  4 |__6|___|___|___|   C4), black captures white as 12/6=2, the
                        number of intervening squares.

    The Power             When a number is equal to one of the powers
                          or roots of an opposing pawn, the latter
can be taken on condition that the capturing pawn be within a
Regular move of the victim.
      A   B   C   D
     _______________
  1 |___|__3|___|___|   B1: Round white  B3: Square black
  2 |___|___|___|___|
  3 |___|_81|___|___|   If the 3 is moved to A2 or C2, it can take
  4 |___|___|___|___|   81 as 3 is the fourth root of 81.

    The Progression       When a number can be made part of an
                          Arithmetic, Geometric, or Harmonic
progression with at least two opposing pawns, it can be taken on
condition that both capturing pawns be within a Regular move of
the victim.
      A   B   C   D
     _______________
  1 |_25|___|___|_15|   A1: Round white  D1: Square black  C3: Tri. black
  2 |___|___|___|___|
```

```
3 |___|___|_20|___|   If black moves his 20 to A3, he can take the
4 |___|___|___|___|   25 as 15-20-25 is an Arithmetic progression
                      (of ratio 5) and both his 15 and 20 will be
within a Regular move of the 25.
```

The Imprisonment — If a pawn is so surrounded that it cannot accomplish a Regular move, it can be captured.

```
    A   B   C   D
   _______________
1 |___|__2|___|___|   B2: Tri. white  D4: Tri. black
2 |_15|_30|___|_36|
3 |__4|___|___|___|   If black moves his 72 to B4, he imprisons
4 |___|___|___|_72|   the white 30 as all of his possible Regular
                      moves will then be blocked by black or white
                      pawns.
```

Pyramids
========

Pyramids can be taken apart one component pawn at a time, or all at once. A pyramid's value is given, at all times, by the sum of the values of its component pieces. The pyramid can itself capture, using either its total value, or the value of any one of its component pieces. The only restriction is that it cannot dislocate itself when moving.

--Urhixidur